



Groupe Consultatif

Internal Models

Draft, 19 November 2008

“Internal Models”

The Draft Framework Directive¹ (FD) has set out the principles of Solvency II. It outlines an economic and risk sensitive solvency framework for the European insurance industry. It states that “the main objective of (re)insurance regulation and supervision is adequate policyholder protection. Other objectives such as financial stability and fair and stable markets should also be taken into account but should not undermine that main objective”, which is to be kept in mind when reading this paper.

The FD refers in several places to “models” be it for assessing technical provisions, determining the value of assets and liabilities and thereby basic own funds or for determining the Solvency Capital Requirement (SCR). It stipulates that approval processes for the use of internal models are to be defined, requiring procedures and criteria for validating models.

The scope of and methods applied in models are constantly evolving and therefore it is important that any model methodology allows for development in the future. This paper aims at suggesting an approach that will facilitate supervisory validation of models, both current and future.

Supporting the validation of models and in particular the validation of “the internal model” used to determine the SCR, is one of the foremost goals of this paper. Validation can be seen as a mutual learning process and the role that models play in an orderly risk dialogue between supervisor and insurer cannot be overstated in this respect.

This paper will therefore propose specific criteria and procedures that supervisors can employ for the “validation” of models, i.e. for deciding between appropriate and inappropriate models. In order to achieve this, the paper sets out to establish explicit principles rather than simple quantitative rules which might only poorly capture future situations.

However, a constraint will be that these principles are unambiguous enough that a trained professional, e.g. an actuary, must be able to understand, interpret and implement them. The paper will focus on the role of professional judgement in this context. It should be noted that the FD already follows a principles based approach and the most fundamental principles underlying modelling in Solvency II are already stated there.

This paper supports the above stated goals from a specific actuarial perspective. Actuaries have long standing experience in designing, using and managing models used in social and private insurance, including pensions. Actuarial science as an activity applies mathematics, economics, law and other sciences to insurance.

The paper aims to

1. describe the objectives following the principles and goals aimed at by the FD,
2. define the methodology used in modelling,

¹ COMMISSION OF THE EUROPEAN COMMUNITIES: *Amended Proposal for a DIRECTIVE OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL on the taking-up and pursuit of the business of Insurance and Reinsurance*, Brussels, 26.2.2008, COM(2008) 119 final, 2007/0143 (COD).

3. formulate and explain concepts, terms and definitions necessary for designing, implementing and using models,
4. describe some of the models and modelling approaches used in insurance and possibly other financial services industries,
5. outline possible internal modelling frameworks and the modelling processes,
6. give advice on concrete and proportionate implementation approaches, and
7. make proposals for useful criteria, limits and constraints for the validation and approval of internal models.

Table of contents

“Internal Models”	2
Table of contents.....	4
1. The role of models in Solvency II.....	6
2. Components of “the Internal Model”	7
2.1 The risk factor model.....	7
2.2 The liability model.....	8
2.2.1. Non-life liability modelling	8
2.2.2. Life liability modelling.....	8
2.2.3. The asset model	9
2.2.4. The management model.....	9
2.2.5. The reporting model.....	9
2.2.6. The competitor behaviour model in the liability market.....	9
2.3 Aggregation	9
2.3.1. Modular approach to aggregation.....	9
2.3.2. Full model approach to aggregation	10
3. Validation of internal models	11
3.1 Professional judgement	11
3.2 Characteristics of professional judgement.....	12
3.3 Problem areas requiring actuarial professional judgement	12
3.4 Transparency and disclosure	13
3.5 Approving the use of internal models	14
3.5.1. Documentation standards (Art. 123).....	14
3.5.2. Use test (Art. 118).....	16
3.5.3. Statistical quality standards (Art. 119)	17
3.5.4. Calibration Standards Art. 120)	17
3.5.5. Profit and loss attribution (Art. 121)	18
3.5.6. Validation standards (Art. 122)	18
4. Models in insurance – definitions	19
4.1 Approach to modelling – “model methodology”	19

4.2	Models	19
4.3	Internal and external models	19
4.4	Conceptual models.....	20
4.4.1.	Description.....	20
4.4.2.	Measurement standard.....	20
4.4.3.	Objective.....	20
4.4.4.	Causality - “risk factors” and “exposure”.....	21
4.4.5.	Objectively observable risk factors	21
5.	Value Measurement Framework.....	22
5.1	Objectives of the valuation framework.....	23
5.2	Financial market consistent valuation standard.....	23
5.3	The market-consistent value of cash flows.....	24
5.3.1.	Valuing the cash flows of a traded contract.....	24
5.3.2.	Valuing the cash flows of a not liquidly traded contract - replication concept and procedure	25
5.3.3.	Risk neutral measure.....	26
5.4	Determining the contracts’ cash flows – “perspective dependence”	29
5.5	Example 1 - The replication of certain cash flows	30
5.6	Example 2 – Value of distributions of a percentage of a fund	30
6.	Risk measurement framework	32
6.1	Risk and uncertainty	32
6.2	The insurer’s risk	32
6.3	The insurer’s exposure to risk	33
6.4	The insurer’s internal risk model.....	34
6.5	The risk measure.....	34
6.6	Criteria for appropriateness of risk measurement	35
6.6.1.	Exposure.....	35
6.6.2.	Time Horizon	36
6.6.3.	Going-Concern vs. Run-Off	37
6.6.4.	Risk Measure.....	38
6.6.5.	Dependencies.....	38

1. The role of models in Solvency II

The FD sets standards and rules insurers must meet if they want to transact business in Europe. To achieve the goals of Solvency II the FD describes quantitative, qualitative and disclosure requirements. First and foremost the FD sets out quantitative rules on the financial resources, which a (re-)insurer must observe, in order to achieve Solvency II's main goal of adequate policyholder protection.

In doing so the FD describes the principles and rules which must underlie the determination of some key quantities, like available capital (own funds and other risk bearing elements) and required capital (SCR). One important role of models is to adequately compute the required quantities according to the stated principles and rules.

Where standardised approaches, like the "Proxy Solutions" for technical provisions or the "Standard Formula" for the SCR are deemed inadequate or where an insurer so chooses, insurers will have to use individually designed and implemented models. The FD requires supervisors to examine these "internal models" and to decide on their appropriateness by approving their use. According to the FD a model can be considered appropriate, if it accomplishes all relevant modelling objectives.

2. Components of “the Internal Model”

As will be described in more detail later in this paper, a typical insurance company will design, build and use a multitude of models.

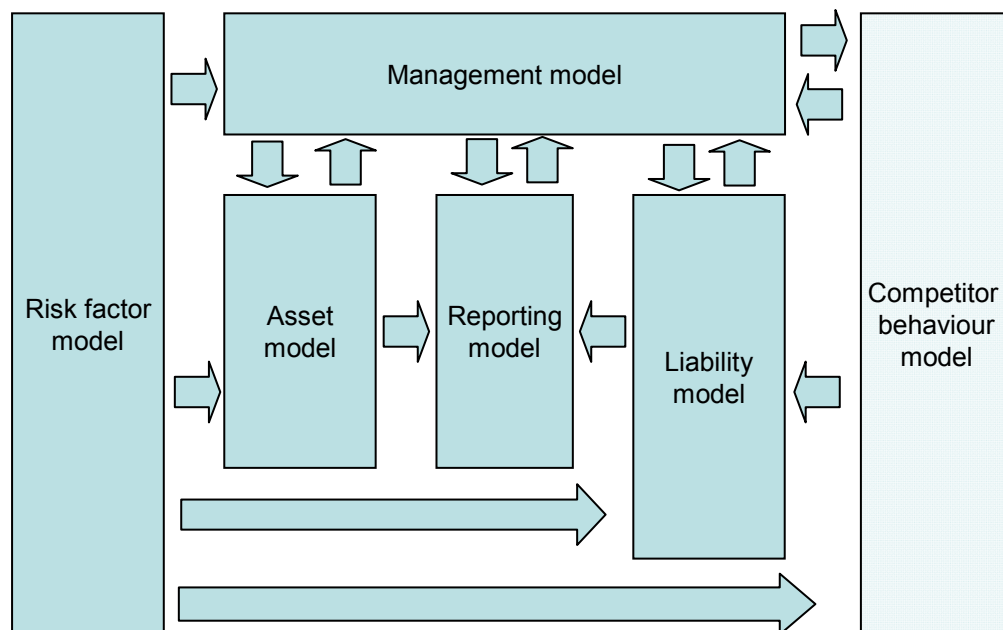
The set of models used to determine capital adequacy and to determine solvency, as defined by Solvency II, are often referred to as “the Internal Model”.

The CEA - Groupe Consultatif Glossary on insurance terms defines the internal model as

“Risk management system of an insurer for the analysis of the overall risk situation of the insurance undertaking, to quantify risks and/or to determine the capital requirement on the basis of the company specific risk profile.”

We consider that the standards and processes used to value all assets and liabilities are integral parts of the internal model, as capital requirements have to be compared with the available capital resources to provide a meaningful analysis of the overall risk situation.

From a high level perspective, the internal model could be split into various components. These components can aid discussions around the process of establishing the internal model. One such example is given in the diagram below and described further in this section.



2.1 The risk factor model

This model will usually provide scenarios for yield curves, equity indices, credit spreads, credit defaults, inflation and for other relevant sources of risk from the simulated environment such as mortality rates, persistency levels, natural catastrophes, operational risks etc.

One common example of this model for the economic environment is an Economic Scenario Generator (ESG). As for all the other components, this model needs a meth-

odological architecture and a parameter set which reflects current conditions. In practice some of the risk factors might be modelled in the liability models.

2.2 The liability model

This model will describe all of the relevant features of the insurance business and their exposure to the risk factors. This will include the necessary information to project the in-force business cash flows, commissions, expenses, reinsurance premiums and recoverables and technical provisions and may include new business.

2.2.1. Non-life liability modelling

- Two "different" models are usually included: premium model and reserving model.
- The premium model includes only new business and reflects the risk that premiums are not sufficient to meet claims and operational expenses. Usually total expenses are split into attritional, large and catastrophic.
- Premium models: a frequency-severity approach (collective risk theory model), natural catastrophe models, assigned distribution for aggregated claims (short-cut methods) are used.
- Reserve risk can be modelled in several ways (e.g. analytical and stochastic simulation). The risk modelled is that, in the chosen time horizon, the company is not able to meet its obligations. The payment pattern is one of the key characteristics of the reserve model.
- Reserve models: Mack, Bootstrap, stochastic chain ladder etc.
- The portfolio can be split into different lines of business or geographical areas which share the same set of characteristics (risk profiles).
- In some cases a scenario based approach can be used (e.g. for NatCat, Large losses), although this may be part of the risk factor model.

2.2.2. Life liability modelling

- The liability model will usually take as inputs the risk factors which influence the development of the cash flows e.g. mortality rates, persistency rates, expense inflation and disability rates.
- The liability model will usually reflect all major lines of business and capture all relevant options and guarantees.
- The liability model will interact with the management model where management discretion exists in the development of the cash flows e.g. in setting bonuses/crediting rates, surrender value bases, new business premiums and charges.
- The liability model should reflect dynamic policyholder behaviour where relevant e.g. in relation to the exercise of options such as guaranteed minimum withdrawals. This may require interaction with the competitor behaviour model.

- The liability model will take as input detailed policy level data, although this will frequently be summarised/grouped in the form of model points (to accelerate model run times).

2.2.3. The asset model

This model will describe all of the relevant features of the investment portfolio. This will include the necessary information to project the asset cash flows, investment expenses and values (market values, book values etc) and may include assets that are assumed to be purchased in the future.

The asset model will usually reflect all major asset classes held (or likely to be held) e.g. cash, fixed interest government bonds, inflation-linked government bonds, corporate bonds, equities, real estate, mortgages and simple derivatives.

The asset model will take as input detailed, individual asset level data, although this will frequently be grouped in the form of model points (to accelerate model run times or to simplify the modelling). These model points should capture the material features of the assets held.

The asset model will interact with the management model to determine which assets are bought and sold.

2.2.4. The management model

The management model governs the interactions between the asset model and the liability model. Typical management actions modelled include the choice of assets to buy or sell (e.g. to meet a need for cash to pay a claim, to manage the book value return or to manage market risk), the choice of bonus to declare, the level of new business premiums and variations to charges or surrender value bases.

2.2.5. The reporting model

This model collects all figures from the other models and produces reports for the management and/or the user.

2.2.6. The competitor behaviour model in the liability market

In some internal models there is an additional component to reflect the dynamics of the competitive environment and the interaction with the insurer's management decisions. This may attempt to capture the premium rates set, new money rates offered or bonus levels declared by competitors and feed into the liability model to affect the model of the insurer's own policyholders' behaviour.

2.3 Aggregation

Reflecting the effects of all of the risk factors to which the insurance company is exposed requires a methodology for combining or aggregating results. Some issues to be considered include, whether a full model or modular approach is adopted.

2.3.1. Modular approach to aggregation

Aggregation can be carried out at more than one level in the modular approach.

Aggregation methods can be:

- analytical: var-covar (one has to be careful because of the shape of the marginal distributions); or

- stochastic/simulation based: e.g. copula (Gaussian, Gumbel etc.).

It needs to be considered that

- individual risk distributions (marginal distributions) together with their correlations do not always completely/uniquely determine the multivariate risk distribution (e.g. in the case of Gumbel-copula);
- aggregation of risks usually leads to a reduction of capital charge depending on risk measure, aggregation method, dependencies, size and how granularly risks are split;
- the dependencies between risk factors may not be the same as the dependencies between the capital required for the risks as the “risk exposure function” may not be linear and may include cross-terms (for example a life insurer’s unit linked business with minimum maturity guarantees may be exposed to decreases in persistency in normal market conditions and to declines in the equity markets, but is exposed to increased persistency when equity markets decline significantly).

2.3.2. Full model approach to aggregation

In a full model approach to aggregation there is an integrated risk factor model, which models all risks and their dependencies at the risk factor level, together with exposure models, which express how the risk factors impact on the firm’s value.

The approach to risk factor models is the same as described in the modular approach: The joint distribution of the risk factors at a given time is arrived at from the individual, marginal distributions by gluing them together, possibly through the use of an appropriate copula.

Exposures created by portfolios are represented by functions of the risk factors, which can, for example, be chosen to produce the value changes since last reporting date. Aggregation, i.e. determining the joint effect the risk factors can have on the value of the firm, given its investment and insurance portfolios, amounts then to summing all these functions. The probability distribution of the sum finally serves to determine economic capital and SCR, respectively.

3. Validation of internal models

All insurance firms using models need to gain and keep control over the quality and fitness for purpose of their models. Supervisors in the European Union will be required by the emerging Solvency II laws and regulations to approve the use of those internal models used to determine the solvency relevant key quantities. In both cases there is clearly a need for governance processes assuring the validity of the models used.

There is a widely held view that the features of models alone cannot assure the proper use and functioning of models. There is also no possibility to prove models correct. Requirements can better be defined on the quality, knowledge and skills of the people involved in insurance business. What one can expect from model use is that knowledgeable professionals are involved in their design, implementation and use and that independent professionals agree and confirm that the models and the model processes are adequate and appropriate for the purpose.

For all types of models and all phases of their development, validation and approval processes professional judgement is required. In the sequel professional judgement will be defined and it will be shown that it is indispensable for internal or external model use approval. It will be shown indispensable on the approver as well as on the applicant side. Documentation and sharing of data, methodology and processes enables scrutiny by professionals within one's own firm, within supervisory authorities and the public. Appropriate and adequate disclosure is therefore required and it is aimed at creating the necessary transparency, which are equally characteristic of scientific review and validation processes.

For supervisory model use approval the FD envisages specific standards and tests, which are outlined in articles 118 to 124. We will explain that these pertain to the judgement expected from insurance professionals and actuaries in particular and that these are specifically trained for undertaking the necessary analysis and for taking decisions in these areas.

3.1 Professional judgement

We define a professional as a person who is familiar with the language, the knowledge and the tradition of a particular trade. A judgement is generally a decision, which is underlined with a justification. Our interest will be in insurance professionals, i.e. professionals active in insurance or an insurance related area. In this context a judgement is a decision taken concerning some insurance question together with appropriate reasoning.

Professional judgement therefore quite generally refers to those substantiated decisions a professional makes in his or her² respective area of competence, which are based on the professional's knowledge of the trade and her problem solving skills.

Actuarial judgement relates to the decisions, solutions and answers by insurance professionals, e.g. actuaries, when questions or issues arising in insurance and pension business, particularly of a quantitative nature, need a resolution. Actuarial judgement has to be accompanied by the reasons and deliberations that led to the judgement.

² We will henceforth use the female gender when referring to a professional, but by no means should this be interpreted as a prejudice against male professionals.

A negative description of what actuarial judgement should not be seems to be in place. Actuarial judgement is neither fortune telling, replacing facts by fiction nor taking arbitrary decisions. Situations where no reasonable statements can be made, no conclusions be drawn or no quantification be made are traditionally regarded as uninsurable and actuarial judgement requires to state so clearly.

Professional judgement requires a professional environment and professional standards. Far from asking for codifying powers for actuarial associations and trade bodies, there still is a need for actuarial associations to play a role as an authoritative body to issue actuarial standards and to give advice on topical issues. We would be happy to see actuarial associations in a supportive role for the active insurance professionals in generally arising insurance issues, but not in an innovation constraining oversight role.

3.2 Characteristics of professional judgement³

In general, a professional applies rigorous analysis to arrive at judgements. In whatever area of activity, she will consider all available knowledge, facts, data and other available information. This includes that she also considers solutions her profession has chosen in the past in comparable situations. To arrive at conclusions she applies reasoning specific to her area of competence and presents corroborating evidence of the points in question. In reality often seemingly contradicting views, opinions and theories exist. The professional weighs the various diverging parts and balances the pros and cons, before coming up with her own judgement.

Most importantly by documenting and sharing all methodology, assumptions and data she makes her findings available for scrutiny by other professionals. There are also approaches to commonly occurring insurance issues and problems which are worth to describe.

3.3 Problem areas requiring actuarial professional judgement

Following the previous definitions actuarial judgement is judgement by insurance professionals, particularly when deciding on quantitative questions. Actuaries are insurance professionals and are typically involved in modelling activities, be it for pricing insurance products, valuing assets and liabilities or for measuring risk.

Some areas are especially in need of professional judgement, but there are hardly any quantitative insurance activities which would not require professional judgement.

Applying principles, for example those of Solvency II, requires more professional judgement than say mechanistic rules, as might be found in Solvency I. The recent CEIOPS *Report on issues regarding the valuation of structured credit products* has pointed out that a “principle-based set of accounting rules must inevitably rely on the fact that answers to such questions are derived from the professional judgment exercised by the individual preparers of accounts.” Quite generally, when a goal is set out, but no prescription for how to reach it, an actuary’s experience and knowledge, i.e. her actuarial judgement, is required to devise the steps to solve the problem. Obviously, the steps to solve a problem are not unique, but on the basis of her education and ex-

³ Because describing professional judgement is very close to describing the „scientific method“, it is no coincidence that the Wikipedia lists similar characteristics for the scientific method as we do for professional judgement, including the so often mentioned importance of “full disclosure” to enable market discipline and public scrutiny, respectively.

perience the actuary should be in a position to judge whether a solution exists, can be implemented and whether it achieves the given objective.

Using statistics and probability: most quantitative questions in insurance require the application of methods described in probability theory and statistics. Actuarial judgement is indispensable for their proper application. As an additional benefit to being able to understand and use statistical methods, the actuary can properly judge their limits, limits in the meaning of credibility, confidence or applicability. These mathematical theories are in particular applied in situations where data is incomplete or the quality of data is poor, often by the nature of the problem. For example, an insurer will never have all the information about a car driver to make definite forecasts about his or her future driving quality. Another example: there will never be enough observations to determine the entire probability distribution of some observable with certainty, but there are statistical methods for optimal estimates. A last example: technical provisions for claims in some lines of insurance, e.g. third party liability, would be grossly understated if only known claims were reserved for. A large body of literature deals with actuarial methods for how to overcome this situation of incomplete data for claims incurred, but not yet reported.

Forecasting based on knowledge of the past: in almost all quantitative activities forecasting plays a major role in the models used in insurance. Judgement has to be applied as to the extent past experience is indicative of the future evolution of some quantity which needs to be projected. As an example, price volatility of equity instruments of a few past days might be indicative of price volatility for the next few days to come in normal market conditions, whereas this observation period hardly suffices for modelling major stock market crashes, which drive capital adequacy requirements. Similarly, some major natural catastrophes might have periods of recurrence in the hundreds of years. Actuarial judgement will exclude drawing conclusions from a few years of observation, but will need to consider, say, geological data ranging back possibly thousands of years.

Using proxies: often if not almost always the trait of interest cannot be observed directly, but the observation of some other trait or the same trait, but within another population is expected to reveal at least some of the missing information. For example, some country might miss information about non-smoker versus smoker mortality. If statistics were available from some other country, they might serve as a first estimate. Another example could be setting risk adjusted premiums via a bonus-malus system for motor policies. It might use an individual driver's past experience as proxy of the driver's accident proneness. Or yet another example, income could serve as proxy to other life style factors impacting on mortality.

Simplifying appropriately: reality is generally complex and insurance would probably not be feasible, if each and every detail of the physical world had to be reflected in actuarial models. Experience shows that the required simplifications are possible, which render insurance modelling tractable, as is manifest in thriving insurance industries. As the previous examples already showed actuarial judgement is required when deciding what is an appropriate simplification, i.e. in deciding which factors are relevant and material and which ones are not.

3.4 Transparency and disclosure

Transparency can be interpreted as the provision of sufficient information to facilitate an understanding of a particular topic or subject. Transparency can be both internal ,

i.e. within the company, among departments, teams, individuals etc., and external, i.e. towards external reviewer, regulator, public etc.

In the context of an internal model, transparency should help facilitate challenge to a model and a widespread understanding of the model's purpose and its strengths and weaknesses.

Disclosure can be seen as mechanism for achieving transparency and care should be taken to ensure that key assumptions underlying the model are highlighted. Excessive or misleading disclosure of information, which obscure the key messages, can be as big an obstacle to transparency as insufficient disclosure.

3.5 Approving the use of internal models

The goal of this section is to describe some criteria which can be used to differentiate between appropriate and inappropriate models. We will link these criteria where possible to the standards mentioned in the draft Framework Directive⁴ in articles 118 to 124. However, the criteria can just as well serve for the internal model validation and reviewing processes.

In short, our advice is to apply professional judgement to judge the firms' professional judgement. The main criterion is consistency. Inner logical consistency of the justification given to the models, consistency with past justifications in the reasoning for changes as well as consistency with actuarial standards and current knowledge.

When designing, implementing and using internal models and when setting objectives for their use no doubt arises that professional judgement is needed. The question arises how to discern proper judgement from improper one. It is not only an issue of supervisory approval of an internal model, but also one of management being assured of the internal models' proper functioning. No certainty can exist about an actuarial model's correctness, in the same way as there is no certainty about laws of nature. However, documentation, review and validation processes analogous to those in science can give satisfactory assurance about the appropriateness of models, even though they give no absolute certainty about their correctness.

For supervisory purposes under Solvency II, articles 110 to 125 give due care to the requirements of internal models' approval processes. It is implicit that the same standards on the ability of professional judgement by supervisory authorities apply as are required on the side of the supervised firms.

We will make reference to the respective Framework Directive articles when discussing the various approval criteria. Their use applies not only to supervisory approval processes, but just as well to internal approval of models.

3.5.1. Documentation standards (Art. 123)

The documentation can be seen as tool to achieve model transparency. The documentation can be used to deliver an adequate understanding of the model for various interested parties. E.g. the documentation for public disclosure will be different to documentation presented to management.

⁴ See reference page 1.

The documentation should be reliable, accurate, appropriate and not misleading. It should cover all areas of internal modelling.

The documentation is typically divided into several areas. This is done on purpose to differentiate the requirements according to the audience it is referring to or assigned to. Setting the documentation and reporting requirements at an appropriate level might require judgement.

The documentation should also contain self-explanatory descriptions for all major and critical areas. This is to be understood that skilled and knowledgeable professionals would be able to replicate these areas based on the documentation.

The procedures used to maintain, update and operate the model should also be an integral part of the documentation. The documentation should be sufficiently detailed to enable a third-party to understand the essential features of the existing model.

The documentation for each internal model can be divided into the following three parts, hereinafter called the „levels“, i.e. general level, level with technical orientation and appendices. These levels are distinguished according to the information's depth.

Level 1 – General: At this first stage, the documentation for each model should include a brief summary that provides a clear description of the purpose of internal model, areas of application of such model, as well as description of the company and its context. An undertaking should also provide details on its risk framework, e.g. risk appetite, financial hedging and other risk mitigation, like reinsurance, IT systems and governance of an internal model. A global view of the first level also justifies the need to utilise an internal model. This is the so-called “use test“ and includes uses in pricing, capital allocation etc.

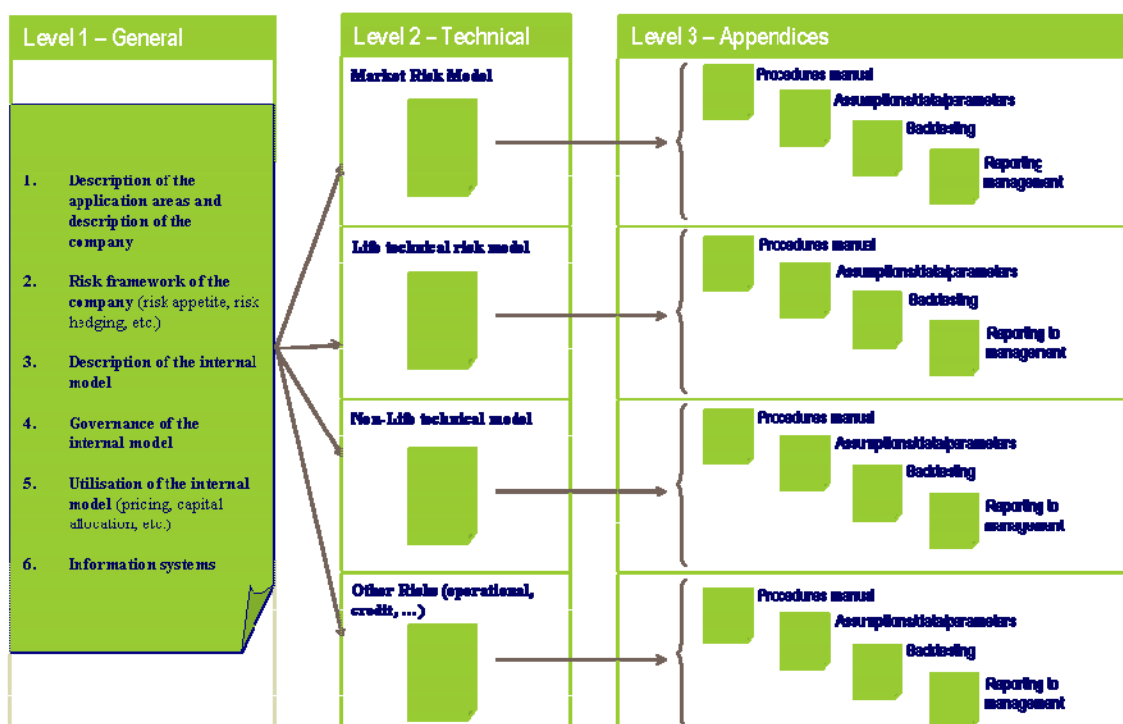
Level 2 – Technical: Higher-level details on the scope and requirements of information presented by the undertakings are the proper disclosure that should be covered by the second level. If appropriate with respect to the information provided on level 1, the core parts mentioned above might be separated and segmented into subsections, e.g. into financial market risk model, life technical risk model, non-life technical risk model, and other risks.

It is currently envisaged that these sections are based on more prescriptive information documentation. It is possible to find key information here of how to use the model and any other technical aspects which can help to understand technical points of view.

Level 3 – Appendices: Model documentation should include in every case detailed appendices. Typically exact implementation details, inclusive formulae, should be included at this level. It should also include a review of the data and assumptions used as an input for a model, parameters already used to make it easy for review of appropriateness and reasonableness of data and assumptions. Validation tests and checks should be also included.

Example Documentation for an Internal Model

Overall framework



Additional to the documentation risk reports from the internal model are generally pre-defined by the management and are generated on regular basis. These reports typically include both qualitative information and quantitative information of the total balance sheet such as SCR, MCR and available capital.

3.5.2. Use test (Art. 118)

A strong indication of properly applied professional judgement is found when the incentives to use the model are aligned with the objectives set by the approver. Art. 118 requires that the model is widely used and plays an important role in the firm's system of governance, in particular the risk management system and the decision-making processes as well as the firm's economic and solvency capital assessment and allocation processes. By requiring wide use the article aims to ensure that risk assessment and management activity take place, improving the firm's risk management abilities and capabilities. The article is not explicit how to align modelling incentives with general stakeholder objectives, but the tone and content of this and the other internal model articles strongly suggest that success in complete and accurate risk assessment is expected to lead to adequate capital allocation and risk limit processes, fulfilling the major policyholder protection objective.

Evidence of the degree to which the use test is met can be gathered from the management information produced, from the metrics used in decision-making and the level of understanding and awareness of the model throughout the organisation. This can be assessed by reviewing management information packs, minutes of committee and board of directors discussions and through interviews with or surveys of individuals in the organisation.

3.5.3. Statistical quality standards (Art. 119)

There is a range of steps that can be taken to assess the robustness of a model. These include:

1. The appropriateness of the data gathered to calibrate the risk factor model, even if the model is an expert assessment of the risk factor, should be assessed – e.g. does the equity index data adequately reflect the underlying equities held, is the “expert” the appropriate expert for the risk being considered?
2. The rationale for any choices made in using the data can be reviewed e.g. choosing the most recent observations only; giving more weight to a period felt to be most relevant to current circumstances; using annual, monthly or daily observations; adjusting the data for example by removing anomalies or errors, or making it more relevant etc. Any choices made are to be tested for bias.
3. If probability distributions have been selected, check which goodness of fit tests have been applied to validate the fitting of the probability distribution of the “as if” data. Quantile-quantile plots may be useful in testing the fit of the distribution to the percentiles of most relevance to the use of the model.
4. Out of sample testing, i.e. where the model is calibrated using a subset of the data and validated against the data not used, may be useful particularly when testing the model fit against less extreme parts of the distribution.
5. If the information is available in comparable formats and corresponds to equivalent risks, it may be possible to benchmark the output of risk factors models for particular selected percentiles and time horizons.
6. It is useful to understand the sensitivity of a model to different choices regarding the use of data and probability distributions. For key risk exposures, it is good practice to produce results based on different data series, for example longer or shorter time periods, different sources etc. This will help illustrate the influence of judgment in the model, may identify whether the choices made introduce a strong bias to decrease the capital required and will give an indication of the possible level of volatility in the model results over time.
7. Risk exposure models may include dynamic models of management actions and policyholder behaviour and may be based on model points. It will be important to test the appropriateness of these simplifying representations of the complex reality. For example for management actions regarding the investment mix a range of different deterministic investment scenarios can be run through the model and the modelled changes in investment mix can be reviewed for reasonableness. Similarly, the cash flows projected from the model points and from the seriatim data can be compared for small number of diverse economic scenarios.

3.5.4. Calibration Standards Art. 120)

An internal model under Solvency 2 should calculate the SCR corresponding to the 99.5% confidence interval or to give policyholders and beneficiaries a level of protection equivalent to this.

It is important to understand that no model can calculate a Solvency Capital Requirement (SCR) that corresponds precisely and uniquely to a given confidence interval. Any

model including the standard approach will be based on a number of assumptions and the end result will never be better than the underlying assumptions made regarding the parameters, distributions and correlations. The capital requirement is simply a product of the assumptions made in respect of the parameters and distributions and of the given confidence level. To discern appropriate from inappropriate assumptions professional judgement is therefore required all the more.

The calibration of the model refers to the process of setting parameters and assumptions that are relevant to the level of confidence one wants to achieve. The testing of the statistical quality of the model and the validation of the model more generally should be appropriate to the target confidence level and time horizon set in the Framework Directive.

3.5.5. Profit and loss attribution (Art. 121)

One specific validation check is the analysis of the profit or loss by source. The sources identified should map closely to the risk factors identified and included in the risk factor model. The unexplained, residual profit or loss should be small otherwise this would be an indication of a significant risk factor that had not been included in the model.

3.5.6. Validation standards (Art. 122)

A check list grown out of practice for standards of validation could look as follows:

1. Check whether the risks identified in the risk register, sometimes called the risk log, correspond to the risk factors modelled. If a risk is not modelled then there should generally be some written justification for this, e.g. that the risk is not material or is considered in aggregate with some other risks.
2. Check whether an appropriate degree of independent challenge and validation of the model has been carried out. This may include documented minutes of a management or administrative body discussion on the model.
3. Appropriate controls around amendments to the model should also be in place.
4. Benchmark portfolios of assets and / or liabilities can be a useful way for removing the effect of variations in risk exposures to compare risk factor models. This can be particularly useful when attempting to compare the equivalence of two models using different confidence levels and time horizons.

However, it should be noted that this would typically require a very detailed specification of the assets and / or liabilities as product specifications and asset characteristics vary widely. Similarly, management decision models and policyholder behaviour models would need to be aligned to increase comparability. A simple example of using a “benchmark portfolio” is running the updated model on the previous asset and liability data to identify separately the effect of changes to the risk factor model.

In the process of defining benchmark portfolios the authorities have to bear in mind not to overburden the companies in respect of the workload.

4. Models in insurance – definitions

This and the following sections give more precise definitions and explanations to the terms used in the previous sections on models and modelling in general.

4.1 Approach to modelling – “model methodology”

Models are simplifications of reality created for some particular purpose. For example reserving models are for determining technical provisions and premium models are for determining premiums. In order to be able to define criteria and procedures to establish which models suffice for their intended purpose and which do not, this paper describes the methods and procedures that allow a rational approach.

Models are used widely in the insurance industry. Experience has shown that the same modelling task can be approached in different ways often without an easy way to compare the different approaches. To enable comparison and to have a solid basis a structured approach is proposed:

1. describe the relevant reality and the desired model output, i.e. the particular purpose the model is going to be used for in plain English, and get agreement on accuracy and appropriateness of description;
2. use a “conceptual model” to capture the formal description of the scope and objectives described in the first bullet point; the required formality of a conceptual model generally leads to more consistency and better insight;
3. compare the actual model which is to be validated with the conceptual model, in which case the conceptual model is also called a “reference model”;
4. define criteria for appropriateness governing allowable deviations from the conceptual model, bearing in mind that the conceptual model is also only a model;
5. describe clearly the required procedures for designing, implementing and using the model including the system infrastructure as well as the knowledge and experience that are required from owners and users of the models and the required accompanying documentation of the models and the procedures.

4.2 Models

Models are used in insurance to describe the economic, business and insurance reality with the appropriate degrees of simplicity and accuracy to achieve a set of objectives. For example pricing and reserving models have long been used in insurance. Examples of more current ones are models for the financial market consistent valuation of all assets and liabilities and the determination of required economic capital.

Less awareness exists that insurers also use other than purely quantitative models. A “business model” or a “bonus policy”, for example, are designed to support an insurer’s decision processes, e.g. to develop, implement and manage its business strategy. In particular, the term “internal model” is sometimes used to describe the business model of a firm.

4.3 Internal and external models

Models in general simplify a complex reality. Models for the same reality and the same objective can differ. The difference might be caused by the different perspectives as-

sumed by the modellers or by different judgement as to which simplifications are appropriate or necessary due to missing information or systems limitations.

In many jurisdictions insurers might have used pricing, reserving or solvency capital models prescribed by regulation in the past and in some places some of these models are still in use. Other institutions, e.g. some rating agencies, also have used models, for example, to measure the financial strength of a firm. Specialised vendor firms sell models for a variety of purposes like “nat cat models”, “economic scenario generators” etc. Many of these models only rely on publicly available information and knowledge. They could be called “external models” as their output can be produced based on externally available information and their parametrisation does not rely on a company’s internal assessment.

Where a model relies on a firm’s internal knowledge, judgement or information that is not available publicly it could be called an “internal model”.

No great importance should be attached to the possible distinction between internal and external models. An insurer might decide to use, say, some rating agency’s capital adequacy model, because it might be deemed sufficiently appropriate. Another insurer might choose the Standard Formula in the future to determine its SCR and economic capital, because of the sufficiently close reflection of this insurer’s underlying risk. In any case the difference is made by the designers, implementers and users of the model and their professional judgement.

The term “the Internal Model” is used in the FD as “internal risk model”, which can offer an alternative to the Standard Formula for calculating SCR, provided its supervisory approval (see CEA – Groupe Consultatif Glossary definition in 2 Components of “the Internal Model”).

4.4 Conceptual models

4.4.1. Description

Models in insurance simplify a possibly complex reality in order to achieve given objectives. A conceptual model describes the concepts of how to create a model which approximates reality and delivers the desired output as accurately as might be wished. This is, of course, limited by current knowledge and viability.

4.4.2. Measurement standard

The current and future states of the world that have to be modelled require the definition of a framework for their measurement. A measurement framework is both a standard and a method with which measurement against the standard can be performed.

For example for Solvency II the most important “states of the world” that an insurer has to model is the evolution of its net worth over time. The corresponding measurement standard is “financial market consistency” and “replication” is a possible method for measurement against the standard (see 5 Value Measurement Framework).

4.4.3. Objective

For any model the desired output must be defined. A premium calculation model, for example, as the name suggests produces premiums, an SCR model seeks to determine required capital and a valuation model determines values of contracts, portfolios and firms.

The objective of a model does not have to be to produce a single number. The purpose of a risk model, for example, can be the production of the probability distribution of the potential changes of the firm's value, leaving the opportunity to choose a risk measure, e.g. one-year 99.5% VaR or 99% TVaR, at a later stage.

4.4.4. Causality - "risk factors" and "exposure"

In a conceptual model, wherever possible and feasible, cause and effect should be separated. This leads to the important distinction between "sources of uncertainty", often called "risk factors", and "exposure", which establishes the link between the risk factors and their impact. In insurance models the link is generally established by contractual relationships, be it between insurer and policyholders or between insurer and companies in which it has invested, and the impact of interest is the change of the value of the insurer.

As an example of this separation between risk factors and exposure take the destruction a European wind storm can cause at different particular locations as the risk factors. Different insurers would be differently impacted by claims. The way and the extent they are impacted reflects their exposures to these risk factors. As another very simple example take the price of a publicly traded stock as the source of uncertainty. The exposure against this risk factor can be directly expressed by the number of shares held.

4.4.5. Objectively observable risk factors

An important consequence of the separation into risk factors and exposure, if it is feasible, is the possibility to identify objectively observable sources of uncertainty. E.g. the price of an investment instrument, say, a government bond, which is traded in a public financial market, can be observed objectively. Designing or choosing a model for the evolution of the price can then take place in public and does not need any company internal information. There are of course many more examples, some of which will be discussed later in the paper.

There are risks where the separation into risk factors and exposure cannot or not so easily be accomplished. For example modelling the risks of third party liability often does not allow the separation. The models then attempt to capture observable phenomena, often relying purely on statistical techniques.

5. Value Measurement Framework

The goal of this section is to describe the value measurement framework underlying economic internal models. A value measurement or valuation attaches a monetary value⁵ to contracts and, more generally, to underlying rights and obligations⁶.

In a monetary economy the exchange of goods and services is based on corresponding exchanges of money. Rights and obligations, e.g. obligations to transfer money in the event of a calamity, can themselves be traded and are so mostly against the exchange of money again.

The definition of value is based on the simple observation that in a monetary economy the rights and obligations to exchange goods and services or to fulfil agreements correspond to, not necessarily simultaneous, exchanges of money. Furthermore, transfer of physical cash (in the form of banknotes or coins) needs to take place, but money transfers can also consist of credits or debits to accounts.

Public and private law and contractual agreements⁷ stipulate the rights and obligations of financial services institutions, in particular of insurance companies towards various parties. Among them are, for example, policyholders and beneficiaries, employees, office building owners, tax authorities, investment instrument issuers and of course the owners of the firm.

These rights and obligations are mostly met by the exchange of money or have - at least in principle - a cash equivalent. As it is customary in economics and actuarial science, we will also adopt the term "cash flows" for these exchanges of money.

The value measurement framework defines the standards of

- how the cash flows arising from rights and obligations are to be determined and
- how a present or current value is assigned to these cash flows.

The framework thereby will define a valuation principle that is comparable with the "replacement cost" or "replacement value" principle used in economics.

⁵ The definition of the meaning of a valuation will only make sense within the scope of a monetary economy. In the possible alternatives to a monetary economy, like subsistence or barter economies, "valuation" would have to define a different measurement standard, like e.g. a certain number of goats as valuation unit, because money as common comparison of value would not be available.

⁶ Contracts are collections of rights and obligations, like for example the right to buy a good, the obligation to deliver a service or the promise to transfer money. The rights and obligations can be implicit, for example based in customarily agreed behaviour, or explicit, e.g. written down in a contractual agreement between various parties.

⁷ The assessment and decision whether something constitutes part of a "contractual agreement" (e.g. whether "policyholder reasonable expectations" constitute an obligation to an insurer), will be left to lawyers and courts, if necessary. Actuaries will need to assess with actuarial judgement which obligations or rights will trigger cash flows as well as their timing and size, with advice from lawyers if appropriate.

5.1 Objectives of the valuation framework

In general, we consider three main objectives to be accomplished with the valuation framework, respectively with the resulting valuations:

- usefulness: valuations should be useful for the intended purpose,
- practicality: valuations should be practical to perform, and
- comparability: valuations, i.e. the resulting values, should be comparable.

It is questionable whether valuations could be useful if they were not comparable with valuations for comparable purposes. Comparability of valuations is aimed for not only within the firm for different areas of activity, but also with other firms' valuations. Even if undervaluations and overvaluations are inevitable as there is no absolute measurement standard, it is important to avoid any systematic or intentional bias, because incentives are then difficult to align with the firm's interests and possibly less profitable projects will be preferred over more profitable ones.

Valuations are performed for many purposes. Owners⁸ and policyholders, but also supervisors and financial analysts, would like to have information about the financial state of the firm, e.g. to support decision making. The management of the firm needs financial information to take decisions be it for example about strategy or about compensation plans. Another purpose might simply be to comply with a law or regulation.

From a Solvency II perspective the valuation framework certainly needs to serve Solvency II's main objective of policyholder protection (see reference in the introduction to the Framework Directive). From the viewpoint of internal models this has consequences for the valuation of the insurer's rights and obligations and consequently for available and required capital resources. These have to be determined in an unbiased and objective way. Internal models have to allow for the realistic assessment of the financial state of the insurer in order to protect policyholders' and beneficiaries' rightful claims.

Therefore the valuation framework has to allow the users of the information, be it the supervisor, owner, policyholder or financial analyst, to assess the insurer's current and expected future financial state. Furthermore the type and quality of the information arising from valuations has to enable regulators to set reliable standards used in the solvency assessment e.g. SCR, MCR, basic and supplementary own funds.

5.2 Financial market consistent valuation standard

In this subsection we aim to define value consistent with the information gathered from financial markets. Standards that rely on consistency with financial markets have been made possible by the huge gains in the storage capacity and computing power of IT systems, but the ideas themselves are much older. One important reason for the preference of defining value relative to prices observed in financial markets is the immediate comparability that is possible by using the same reliable measurement standard.

⁸ We use the term "owner" for "shareholder" in a limited company, "policyholder" of a mutual insurer or "member" of a mutual organisation. Policyholders might therefore have more than one stake in a company, one as holders of an insurance policy and another one as owners of the insurer.

For the purpose of defining a valuation standard it is necessary to define it for all the rights and obligations of a firm. These are characterised by their cash flow pattern. The descriptions of the “cash flow patterns” are possibly complex, because cash flows of unknown size can arise at unknown instances in the future and might depend on optional decisions and other random factors.

For example, let’s look at the description of the cash flow pattern of a public equity investment. Everybody is entitled to buy a publicly traded equity share. If the share is bought, it results in the purchase price as an outflow and the future dividends and other shareholder compensations as uncertain cash inflows until the right to sell the share is executed. Should the owned share be sold, a last inflow of the size of the future share price takes place, and all consecutive cash flows linked to the share cease for the seller. The example should show that in general neither timing, size, sign nor duration into the future of the cash flows is known at the time when the pattern is determined. However, it exemplifies how individual rights and obligations of which a contract consists impact on its cash flows.

As a first conclusion we note that it is sufficient to define the valuation framework by giving a:

- standard for how rights and obligations lead to cash flows and a
- standard for how to assign a value to such cash flow patterns.

We start with the latter first.

5.3 The market-consistent value of cash flows

The following subsections assume that the “cash flows” are given and propose that their value should be determined by replication and the consideration of the cost of holding capital.

5.3.1. Valuing the cash flows of a traded contract

To value a contract⁹, one values the cash flows characteristic of the contract. As one objective for the valuation framework is comparability, the definition of value for this contract has to take into account valuations by other parties. Observable valuations by other parties take place for example, if the same contract is traded¹⁰. This comparability is for example given in the case of a public equity share or an insurance linked security.

Trading a contract requires prices acceptable to both seller and buyer. Assuming that all buyers and sellers act in their best interest and are sufficiently knowledgeable and informed, “market prices” for such contracts (also called instruments) are believed to form a good basis for a valuation standard not only for a firm’s internal purposes, but also for external standards, like for example Solvency II and IFRS.

Some criteria have to be satisfied for such market prices to be best possible estimates of a contract’s value:

⁹ “Contract” is used as generic term for all types of contracts: insurance policy, bond, instrument, security, employment contract etc.

¹⁰ In the next section “same” will be defined as “having the same cash flow pattern”, which is entirely within our view that contracts are characterised by their cash flow patterns.

- the prices have to be objectively observable;
- a sufficient number of the contracts has to be traded for the prices to be reliable;
- the market where the contracts are traded has to allow the challenge of prices, i.e. arbitrageurs have to be able to go long contracts deemed underpriced and to go short contracts deemed overpriced; the increased or decreased demand or supply is believed to have the tendency to eliminate price inefficiencies.¹¹

Definition: Contracts that satisfy these criteria are called “admissible for replication”.

To require these criteria to be satisfied is justified in our valuation context: we aim the value to be a best estimate of the price for which the contracts could be bought or sold. This forces us to

- ensure that the price quotes of the instruments used for replication are reliable and objective, that
- the quantities that have to be bought or sold to pursue some replicating investment strategy (see below) can be absorbed by the market with only small price movements even if others trade at the same time and that
- a large number of market participants regard the quoted prices as the best estimate of the contracts value, which increases the credibility that the prices are good estimates of the values these contracts represent.

Most contracts of an insurer, be they assets or liabilities, do not directly satisfy these criteria. Either 1) they are not traded in a public market, 2) there is no deep and liquid market for them or 3) they are not traded at all. Examples would be 1) many derivatives, e.g. interest rate swaps, are traded “over-the-counter”, 2) private equity and many corporate bonds and 3) most insurance contracts.

5.3.2. Valuing the cash flows of a not liquidly traded contract - replication concept and procedure

In the situation that the contracts to be valued do not satisfy the criteria of the previous subsection, methods and procedures based in financial economics have been developed that can be applied to value the contracts. In fact, the far more fundamental question than “what value should be assigned to contracts” is the one of “how to transfer wealth into the future such that cash is available to meet obligations as and when they fall due”. This question has found a satisfactory answer, which will also illustrate the natural difficulties and limitations of the method.

The basic question about an insurer’s solvency is whether it can honour its promises made towards the various stakeholders. To do so the insurer will have to settle its obligations by payments of cash or something equivalent. To have the financial means available as and when obligations fall due, the insurer will hold and manage investments and cash.

¹¹ Bans on short selling or closing markets as observed again in the recent market turmoil are of course serious impediments to the proper functioning of financial markets described here. The possibility of such frictional events should as a consequence entail accordingly higher capital requirements to buffer the additional risk.

Without any doubt all policyholders' and other creditors' claims are protected with certainty, if the insurer is able to match with certainty the cash outflows that it anticipates will emerge from the obligations, whether the matching is accomplished with cash flows from holding investments or buying or selling instruments or from using the stock of cash.

In reality one needs to relax the requirement that the insurer has to be able to honour all obligations with certainty. An insurer is regarded as being solvent as long as it can be expected to remain able to match all cash flows arising in the future with a high probability. For example, the FD requires that the "success probability" over a one year period to be greater than 99.5%. In practice the insurer can replicate and wants to replicate only to some extent its future cash flows with an accordingly executed investment strategy and use the cash stock for the non-replicated part. Following an investment strategy might well require the insurer to buy and sell investment instruments continuously. In order to be able to determine to what extent the investment strategy replicates the anticipated cash flow pattern, the instruments used in the replication strategy have to be admissible for replication, i.e. satisfy the requirements necessary for reliable financial market consistent valuations explained in the previous section.

Definition: The current value of the cash flows of given contracts is defined to be the current price of the initial portfolio of an optimal replication strategy, adjusted for the cost of holding the stock of cash that acts as a buffer to cover the remaining risk. An optimal replication strategy for given contracts optimally replicates the cash flows arising from the rights and obligations making up these contracts and exists exclusively of instruments admissible for replication. The remaining risk is the non-replicable component of the cash flows. "Optimally" means that any risk that can be eliminated by replication is eliminated.

Remark 1: This general definition for financial market consistent valuations can be applied for any financial instrument or insurance contract. No claim is made though that one will overcome the technical difficulties in all cases and find reliable estimates. Depending on the situation this might even indicate that some insurance cover should be viewed as uninsurable, because no reliable price can be determined, or that a best estimate of an investment has to be re-evaluated. The uncertainty revealed has to be taken into due account in the determination of required capital.

Remark 2: In general an optimal replication will depend on the assumptions made in the "market model", i.e. on the assumptions underlying the modelling of how prices of financial instruments will evolve. We will later give two examples showing that in certain important cases no market model needs to be assumed. One example will be the valuation of certain cash flows, i.e. cash flows where the timing as well as the size of the cash flows are known in advance with certainty. The other example is the valuation of cash outflows to some stakeholder group over time arising from the distribution over time of 10% of a portfolio of investments, where the timing of the distributions is not known.

5.3.3. Risk neutral measure

In this short subsection we link the market-consistent measurement framework to the keywords of the theory and practice of arbitrage pricing known from financial economics. This will give some insight into the modelling ideas and techniques used to implement market-consistent valuations. Thereby the role of risk neutral measures, which are also known as martingale measures in the stochastic process literature, will be ex-

plained. We will use some technical terminology. The purpose is not to develop the theory, but instead simply to give keywords for those wanting to consult the details in the literature.

From the previous sections we need to remember the market-consistent valuation context. We are given cash flows and the task is to find an optimal investment strategy with cash and financial instruments to replicate the given cash flows. The instruments are supposed to be traded liquidly enough for the strategy in an efficient, basically arbitrage-free financial market. Modelling needs to simplify and to remove details of reality that are unimportant for the objectives given. However, with every model we should check whether the parsimony achieved still leaves enough of the richness needed. Since Black-Scholes seminal paper on option pricing, many issues and problems have been clarified, sometimes unfortunately but necessarily in a way that naïve notions had to be replaced with better, but possibly less intuitive and certainly more complicated ones.

We are going to describe concepts outlined above in turn: how “markets” are modelled, how to express their efficiency or arbitrage-freeness, how to single out admissible investment strategies to avoid unrealistic cases, how to characterise when a replication for a given cash flow exists and what it means to have an optimal replication strategy. Last but not least we will mention the research area that looks for practical and effective methods.¹²

5.3.3.1. Representation of the “financial market”

The prices of the traded instruments are modelled as special stochastic processes, called semimartingales (for an intuitive description of stochastic processes see 6.1 Risk and uncertainty). A process in this class allows the definition of stochastic integration with the resulting process again being a semimartingale. The classic example is integration driven by Brownian motion, not only a semi-, but even a full martingale. A Brownian motion with drift no longer is a martingale, but still a semimartingale. In the model world chosen for the market the stochastic price processes are driven by a so called real world probability law, which we denote with P . If one divides all prices by the price of a numeraire instrument, e.g. a risk free currency account, one calls the processes “discounted price processes”.

5.3.3.2. Markets free of arbitrage with admissible investment strategies

We have described in 5.3.1 the real world financial market in a way that an arbitrage possibility, i.e. the possibility to make a profit without risk, if it existed, would disappear quickly, because in open and free markets “speculating traders” are permitted to exploit

¹² Three summary articles accessible on the Internet:

Jia-An Yan, A new look at the fundamental theorem of asset pricing, *J. Korean Math. Soc.* 35 (1998), No. 3, pp. 659-673

Mark H.A. Davis, Martingale Representation and All That (2005) (*in Advances in Control, Communication Networks, and Transportation Systems: In Honor of Pravin Varaiya, E.H. Abed (Ed.), Systems and Control: Foundations and Applications Series, Birkhauser, Boston, 2005*)

Martin Schweizer, A Guided Tour through Quadratic Hedging Approaches, (2001) (*in E. Jouini, J. Cvitanic, M. Musiela (eds.), "Option Pricing, Interest Rates and Risk Management", Cambridge University Press, 538-574*)

any perceived arbitrage opportunities immediately, thereby making them disappear again.

An investment strategy is modelled by stochastic processes expressing the number of units held or owed of cash or positions held or owed of the instruments. Even if it is not possible in general to map all real world constraints that might apply, modelling investment strategies has to at least consider that budget constraints exist, as no one has unlimited credit for example, and that investment decisions have to be taken before trades take place. The desired investment strategy should be “risk free”, which corresponds to the requirement of being “self-financing”. The integration process mentioned in the previous subparagraph is used to express the evolution of the portfolio value over time as the sum of a self-financing part and a remainder, which is zero for self-financing strategies.

The **first fundamental theorem of asset pricing** expresses the finding that:

No “arbitrage profit without risk” or no “free lunch with vanishing risk” is possible if and only if there exists at least one risk neutral, or martingale, measure Q for the discounted price processes, which is equivalent to the real world measure P .

5.3.3.3. Optimal replication strategy

The fundamental paradigm change that took place with Black-Scholes was not the invention of some new calculation device, but the insight that claims can be replicated, i.e. risk can be hedged away, by an appropriately chosen, possibly dynamic, self-financing investment strategy. It is intuitively clear that only claims that are measurable with respect to all information gathered from the prices of the financial instruments, if any, can be replicated. Let F_t denote all market information up to time t . With the early models for the evolution of instrument prices, as e.g. geometric Brownian motions, it was possible to prove the following **second fundamental theorem of asset pricing**:

There exist an admissible dynamic investment strategy from time zero until time T that replicates an F_T -measurable claim, i.e. a claim that can be fully explained with the information coming from the market up to time T , if and only if there exist exactly one martingale measure for the discounted price processes, which is equivalent to the real world measure P .

More general assumptions for the price processes, like allowing jumps or varying volatility, and claims that are no longer F_T -measurable necessitate clarification as to what extent the above “martingale representation” theorem remains true. A particularly beautiful theory was developed for the discounted prices modelled as the special class of L^2 -martingales. Within such a representation a geometric interpretation is given to the “hedgeable component” of given cash flows, namely their orthogonal projection onto the closed subspace of claims attainable with admissible investment strategies. The hedgeable and non-hedgeable components turn out to be “strongly orthogonal”, hence the claim of optimal replication. In the classic theory the method is known as “mean variance hedging”.

5.3.3.4. Effective replication – the Malliavin calculus

The first and second fundamental theorems of asset pricing are for most purposes pure existence theorems. For concrete uses effective and implementable algorithms are needed. Besides general methods and tools known from the theory of stochastic differential equations, effective methods are needed to determine the replicating investment

strategy. Concrete knowledge of the replicating portfolio is for example needed for determining and then managing financial market risk exposures, besides the obvious uses for valuation purposes. The currently most effective methods derive from stochastic calculus of variations or Malliavin calculus applied to Monte Carlo methods.¹³

5.4 Determining the contracts' cash flows – “perspective dependence”

When valuations are used, internally for management or externally for financial disclosure, it is important to know which assumptions underlie them. We will give some, but probably not all, choices that have to be made, before one values the resulting cash flows. Depending on the choices, different cash flows can arise.

The concrete models that life or non-life actuaries employ to determine the probability distribution of one specific cash flow, e.g. the cash flow arising from some non-proportional catastrophe reinsurance agreement, are not described in this subsection, but in the specialised appendix sections to be written later. The result of that kind of modelling is an input for the valuation procedure described here.

- Perspective: whether we determine the cash flows from the perspective of the firm, the policyholders, the owners or of another party.
- Scope: whether the cash flows arising from future new business is or is not taken into account.
- Future states of the firm: whether we assume that the firm remains a going concern or alternatively that it falls into financial distress, an unexpected but possible event.
- State of the world: is the world in a normal or an exceptional state?

Valuations are particularly useful from a firm's perspective, taking only existing business into account. This means one assumes the firm will remain a going concern and one takes only existing business into account. From this starting point adjustments can easily and transparently be made for alternative sets of assumptions.

There will be need to do valuations from, say, a shareholder's perspective. As shareholders have limited liability towards the firm and its clients, the shareholder will want take the possible default of the firm into account. Cash outflows towards clients or creditors will be adjusted accordingly. Also, shareholders would appraise the firm by also valuing anticipated future business.

As a general rule “unexpected situations” should be taken into account in the risk measurement, whereas the expected or anticipated future flows into the value measurement. This applies to the firm specific states as well as for the states of the world. If an insurer, say, intends to fulfill all obligations arising from insurance policies and has all reasons to assume that it will do so, then valuations should assume the firm remains

¹³ The recent progress with respect to concrete and effective application of Malliavin calculus in finance came with work of Pierre-Louis Lions and associates, but the original papers don't seem to be available freely. A textbook reference is:

Paul Malliavin, Anton Thalmaier: Stochastic Calculus of Variations in Mathematical Finance, Birkhäuser, 2006

a going concern. Equally, for valuation purposes the most likely future costs and other factors and parameters are assumed in a valuation¹⁴, because these are part of the “expected” most likely future states.

5.5 Example 1 - The replication of certain cash flows

Let's assume that a series of cash flows is to be valued, where not only the exact times, but also the sizes of the cash flows are assumed to be known with certainty. Let's further assume that there are bonds available in the financial markets that can default neither on interest payments nor on the payment of the principal and that the principal and coupon payments coincide with those of the given cash flows. Bonds issued by many governments are regarded as default risk free.

For example assume cash outflows at $t = 1, 2$ and 3 of 100'000, 200'000 and 50'000 and assume three default risk free bonds exist with maturities at $t = 1, 2$ and 3 , principal amounts each of 1'000 and with annual interest rate coupons of 3%, 3.5% and 4%, respectively.

Finding a portfolio composed of the three bonds together with an investment strategy to replicate the given cash flows amounts to little more than solving a linear equation with three unknown variables, which represent the targeted bond positions. The strategy itself becomes static, i.e. a position remains unchanged until expiry of each bond.

If fractional shares of instruments were possible, holding positions of 88.717 of bond with maturity at $t=1$, 191.379 of bond with maturity at $t=2$ and 48.077 of bond with maturity at $t=3$, i.e. holding a portfolio of default risk free bonds executing a static, passive investment strategy, would produce cash inflows at times $t = 1, 2$ and 3 which exactly offset the given certain cash flows. The resulting cash flow pattern would be nil, indicating total absence of risk. The value of the three cash flows, being cash outflows, therefore is the negative value of the purchase price of the bonds¹⁵.

In reality few insurance cash flows will be certain, but many can be well approximated by certain cash flows. Equally, the default risk free instruments used in the valuation will only approximately suffice for an exact replication.

In order to value the type of certain cash flows in a practical way, discount curves or equivalently yield curves have been constructed for many years. This assumes that constructing a replication for every certain cash flow is within reach using interpolation and smoothing.

5.6 Example 2 – Value of distributions of a percentage of a fund

Assume a portfolio of investment instruments that covers the surplus fund not yet distributed to holders of with-profit savings policies. The insurer's commitment is to distribute 90% to policyholders and 10% to the owners of the firm; that is, the insurer will distribute in due course and spread over time the entire fund and all that accrues from the fund in the future.

¹⁴ This sentence is not meant to be interpreted that the expected value of a function of random variables is the function evaluated at the expected values of these random variables.

¹⁵ To keep the literal meaning of the word „replication“ in order to hedge a risk one would mirror given cash flows whereas to value them one would replicate them, taken the sign conventions correctly into account.

What is the value of the owners' 10% share?

The answer is simple: the present value of 10% of the future distributions amounts to 10% of the current market-consistent value of the portfolio.

Let the instruments ever to be used to replicate this investment portfolio be $P^0, P^1, \dots, P^n, \dots, P^N$ where P^0 denotes the cash in a chosen currency. $P^k(t)$ denotes the price of instrument P^k at time t . The portfolio can be represented as positions $\pi^n(t)$ held at time t of instrument P^n . The various events, like dividend or coupon payments, or investment activities, like buying and selling, will affect the positions without affecting the total value of the portfolio, except for expense and cost accounting. A distribution of surplus affects, say, a fraction α of the portfolio, and will transfer $0.1 \cdot \alpha$ of surplus to the equity account and $0.9 \cdot \alpha$ of surplus in total to individual policyholders' bonus account. Assume that at times $0 < t_1 < t_2 < \dots < t_k < \dots < t_p$ a share $\alpha_1, \alpha_2, \dots, \alpha_k, \dots, \alpha_p$ - each time of the value of the remaining portfolio - is distributed, where $0 < \alpha_k < 1$ for $k=1, \dots, p-1$ and $\alpha_p = 1$. If the value of the portfolio just before a distribution is $V(t_{k-})$, it will amount to $V(t_{k-}) \cdot (1 - \alpha_k)$, i.e.

$$V(t_k) = V(t_{k-}) \cdot (1 - \alpha_k).$$

It is now easy to see, but a bit more difficult to write down nicely, that a portfolio always composed of 10% of the positions $\pi^n(t)$ will exactly replicate the share of the surplus distribution that will go to the owners of the firm. The cash flows corresponding to the distribution to the owners are replicated by always holding 10% of the the exact same portfolio of investment instruments held for the surplus. This implies that all events and management actions which impact on the original portfolio are equally replicated (one might say mimicked).

If the owners of the firm would like to secure the amount of their share of the surplus fund, they would have to borrow the value of it by going 10% of the portfolio short and reduce their short positions over time with each distribution they will be credited with.

A last comment on the example seems necessary. It was clearly necessary that to be able to replicate the cash flows required the ability to trade at reliable prices. So, underlying the claim that the future distributions were 10% of the market-consistent value of the surplus fund was the requirement that it be possible to replicate the distributions with liquidly traded instruments. Otherwise no market-consistent value could have been determined.

6. Risk measurement framework

This section describes a risk measurement framework in general and in the context of Solvency II. It outlines the concepts underlying risk models. Risk models simplify a possibly complex reality (as models do in general). Their objective is to model risk and to measure it. Describing the risk measurement framework requires us to define risk, risk factors, exposure and risk measures in some detail, but still at a conceptual level. The details of concrete implementations of risk models will be dealt with in later sections in appendices, which have not been written yet.

6.1 Risk and uncertainty

In order to take a practical approach to the definition of risk, uncertainty and other concepts linked to them, we deviate from Frank Knight's definitions for risk and uncertainty^{16 17}. We characterise uncertainty as absence of certainty and risk as uncertainty about the outcome of something anticipated or expected. We link quantifiability or the degree of quantifiability to our more or less complete knowledge of the world.

An object's risk is described by the possibly uncertain evolution of its states. Risk therefore is represented by functions which map instances in some time domain into some codomain of attribute values which characterise the states. Certainty of the evolution would be expressed by the fact that only one such function is needed to describe the future states. The mathematical description of uncertainty uses sets of such functions expressing the potentially different future evolutions of states. To become tractable some technical assumptions are needed, but we will not go into details¹⁸ here. The sets of functions are called stochastic processes¹⁹. We also call them risk factors, risk processes, sources of uncertainty and also observables or observable risks, depending on context. "Events" are defined as certain "measurable" subsets. These can be assigned probabilities, which in turn define the probability laws for the stochastic processes. For our purposes it is enough to know that this is possible and can reflect our understanding of the world and its evolution.

6.2 The insurer's risk

In Solvency II the risk to be measured is defined as the unexpected and unanticipated negative change of the insurer's value over a time horizon of one year. The risk process underlying this calculation is the value of the firm. In Solvency II the value of a firm is defined as the value of its "basic own funds". The risk process to be modelled therefore is the evolution of the net worth of the firm and the risk to be measured is its unexpected decrease over the course of one year.

¹⁶ Knight, Frank H., *Risk, Uncertainty, and Profit*. Hart, Schaffner & Marx; Houghton Mifflin Company. 1921. Library of Economics and Liberty.

¹⁷ Frank Knight differentiated between unquantifiable and quantifiable uncertainty and defined the latter as risk.

¹⁸ See any text book on stochastic processes, e.g. Delacherie, C., Meyer, P. A., *Probabilités et potentiel*. Paris: Hermann 1980.

¹⁹ The usefulness of stochastic processes for actuarial considerations is strongly promoted in: A. J. G. Cairns et al., *Stochastic Processes: Learning the Language*. Faculty of Actuaries Students' Society 1998.

For a firm's internal risk management purposes it is useful to consider also alternative definitions for the risk measure, e.g.: The firm's risk to measure is the unexpected shortfall of basic own funds in one year's time, shortfall being defined as the missing amount of basic own funds which allow the firm to continue business as anticipated and planned²⁰ (rather than to simply have positive basic own funds).

In this alternative definition, the firm's risk would thus be measured against the basic own funds that would have to be available at the beginning of the second year. This alternative definition would, for example, address the different objectives of different stakeholders or other interested parties. Solvency II regulators require measurement against own funds needed to run-off business in the books at the end of year one, i.e. Solvency II regulators require capital to buffer "reserve risk" and risk of non-expired covers (part of "premium risk"). Firm owners would in addition plan for new business after one year (which also creates "premium risk") and would take that into account when determining required own funds at the beginning of year two.

If one assumes more or less stationary business, technical provisions, unexpired covers and expected new business are the same at the beginning of the first and second year. Shortfall of basic own funds can therefore be determined against required capital at the beginning of the first or the second year, because they are assumed to be the same. This would bring us back to Solvency II's definition. One advantage among many of the alternative risk definition is that it can potentially help to alleviate concerns about the scaling of risk measurement over a one year period for multi-year business, because by the recursive nature of the definition the whole future evolution and not only its projection as assessed from within the first year would have to be taken into account not only for value, but also risk measurement.

6.3 The insurer's exposure to risk

The previous sections dealt with the definition of risk factors in general and one particular risk process, the firm's net worth. Exposure is defined as the link between them. Exposure thus expresses how risk factors impact on the firm's net worth. For example an insurer's sold products, that is, its written insurance contracts, give cover against specified risks. The insurer is exposed to these risks as their evolution and materialisation impact on the firm's value. In general an insurer will have material exposure only to a few out of a myriad of potential risk factors.

In simple cases exposure can be represented simply. Take for example an equity share in some limited company. The price evolution of the share can be defined as a risk factor. Exposure is created by holding such shares as investments. It can be represented as the number of equity shares held, often termed the investment "position".

As another example take wind storms. We could take cumulative damages per square kilometre as risk factors. The location of the objects covered by the insurer and the form of cover granted against wind storm damages determines the insurer's exposure

²⁰ In economic and business reality the objective of firms and therefore their risk seems to be captured well with this risk definition.

against the wind storm risk factors²¹, i.e. this is how the link of wind storm damages and changes in the insurers net worth could be expressed.

The latter example can also serve for an intuitive understanding that risk factors can more or less be mutually dependent. One can expect some synchronicity of accumulated damages in neighbouring areas, whereas wind storm damages in areas far apart are expected to show less “correlation”.

6.4 The insurer’s internal risk model

We bring together the previous sections on the insurer’s risk. We can state that in order to measure a firm’s risk we need a model for the potential change in economic net worth of the firm. In essence it comprises several elements:

- Models for the various risk factors and their dependencies that the firm can be exposed to; this includes models of the evolution of potential states of the world over time or possibly just for a specific future instance of time;
- exposure models for the firm’s exposure to risk factors; implicit and explicit contractual relationships cause the exposure of the firm’s net worth to the risk factors;
- valuation models that determine the economic net worth of the firm or the change thereof for the various states of the world at future given times.

Value measurement standards must therefore be an integral part of any internal risk model, in particular any economic capital model.

6.5 The risk measure

So far the firm’s risk and the elements needed to determine it, have been defined but no measure has been selected. In theory many different risk measures²², coherent or not, are available and an insurer might chose different risk measures than the one required by Solvency II for its internal risk measurement.

Solvency II requires an insurer to have own funds in excess of the Solvency Capital Requirement (SCR). SCR is defined as the decrease of basic own funds over a one year period, which is exceeded with only 0.5% probability (Article 101, Paragraph 3):

- “The Solvency Capital Requirement shall be calibrated so as to ensure that all quantifiable risks which an insurance or reinsurance undertaking is exposed to are taken into account. With respect to existing business, it shall cover unexpected losses.
- It shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period.”

²¹ Current event-based approaches to catastrophe modelling take an even more fundamental approach to the risk factor definition and exposure is based on many more parameters than just the type of the cover.

²² Philippe Artzner, Freddy Delbaen, Jean-Marc Eber and David Heath: *Coherent Measures of Risk*, Math. Finance 9 (1999), no. 3, 203-228.

The insurer therefore has to calibrate its risk measurement for Solvency II such that the resulting measure determines an amount of economic capital which allows it to buffer all risk during a one year period with 99.5% certainty. To be able to measure risk the internal model projects the states of the world to the end of the one year time horizon. For each potential state of the world, the firm's economic net worth is determined. This allows one to determine the probability distribution of the firm's economic net worth and consequently the SCR²³.

6.6 Criteria for appropriateness of risk measurement

The risk model that is used to measure risk has to consider all risk factors which have an effect on the firm's economic net worth. This is a consequence of the "total balance sheet approach". Those that are found material are called the relevant risk factors.

The risk factors can be split into objectively observable and company specific ones. Objective risk factors are those that can be observed objectively and which essentially cannot be influenced by an individual firm, e.g. interest rates, equity prices or natural catastrophes that define the external state of the world in which the firm operates. Firm specific risk factors are those that the insurer has a decisive influence on. Examples would be the firm's management decisions impacting on sales channels, underwriting practice, claims costs, salaries, rent, and - via diversification of the portfolio - cost of capital etc.

As exposure is firm specific, the relevant risk factors will be firm-dependent. When the insurer develops or updates the internal risk model it has to assess to which risk factors material exposure does and does not exist, i.e. it has to determine the relevant risk factors.

It is important to emphasise that once the valuation framework, the time horizon for risk measurement and the risk factor and exposure models have been determined, the risk is uniquely defined. There is no more choice regarding an "extra" calibration of risk factors. For example, in Solvency II the risk to be measured is the potential change in economic net worth over a one year time horizon. This requires that the change of the risk factors over the same one year time horizon and their impact on the firm's net worth is to be measured. Possible long-term mean reversion of equity prices and other similar assumptions are completely irrelevant for the measurement of risk over the one year period.

The risk model has to have the capacity to project the potential evolution of the risk factors over the given time horizon. Since many risk factors are stochastically dependent among each other, the risk model has to capture these dependencies.

There is no a priori decision procedure which risk factors are relevant and which risk factors will be of lesser materiality. It will be necessary to apply professional judgement in all cases.

6.6.1. Exposure

When determining the firm's exposure to the various risk factors which can have a material impact on it, all relevant contractual relationships, be they explicit or implicit, need

²³ It should not be taken literally that the entire probability distribution has to be determined. Mathematical methods might allow one to determine the 0.5% level or to give upper bounds by other means than by producing the entire distribution.

to be taken into account. This follows from the “total balance sheet approach”, which stipulates that any risk and any exposure that can have an impact on the firm’s economic net worth be taken into account.

Special contractual agreements are reinsurance or derivatives used to manage risk. In particular, if the insurer uses insurance risk mitigation instruments, e.g. reinsurance or securitisation, or financial risk mitigation instruments, e.g. swaps or other derivatives or if the firm is an insurance group and uses intra-group capital and risk transfer instruments, the exposure to various risk factors is affected and the exposure models have to take account of it appropriately

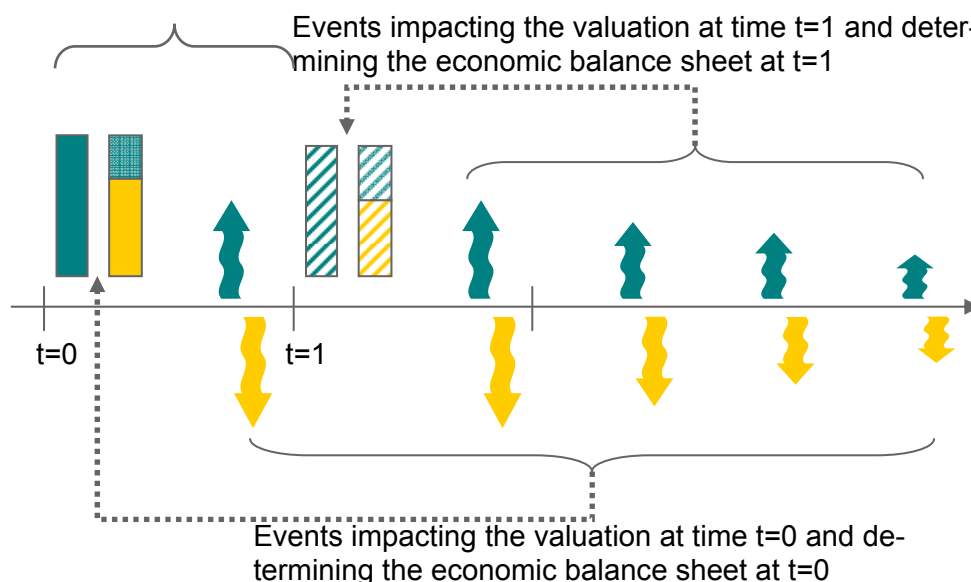
6.6.2. Time Horizon

In the Solvency II risk definition and in many economic capital models, often two basic time horizons are differentiated:

- The time horizon used in the measurement of risk which determines required risk capital, e.g. SCR in Solvency II, and
- the time horizon intrinsic for the valuation of the firm’s assets and liabilities, which determines available capital, e.g. basic own funds in Solvency II, is defined until natural expiry.

Risk capital represents a buffer against unexpected losses during the specified time horizon. In Solvency II and in many economic capital models used by insurers and banks, the time horizon is one year. Aside from the importance of the calendar year and one year financial reporting periodicity in many jurisdictions, the one year horizon for the calibration of risk measurement seems to reflect a reasonable time span to recapitalise a firm if needed after financial distress.

Time horizon for which capital has to be available



By definition the required risk capital represents a buffer against all risks²⁴ that can become manifest during the one year time horizon at a high confidence level. This includes risk from exposures to risk factors such as asset prices, interest rates, volatilities, natural catastrophes, or new information about, say, mortality trends. The latter risk is a typical example where during the one year period the potential for an assumption about the evolution of risk factors beyond the one year time horizon to change has a definite impact on measured risk and therefore required capital, even though no claims' payments must fall within the one year period.

The time horizon for valuation cannot be chosen freely, but is given by the time assets and liabilities are held by the insurer, because all potential future cash flows need to be brought into the valuation, inclusive potential cash flows from sales (as explained in the section on "value measurement").

Risk beyond the time horizon for risk measurement still has an impact on the present. The market consistent value of assets and liabilities consists of two components, one being the best estimate of the replicatable component and the other being the cost for future required capital needed to cover the non-replicatable component of the future cash flows. This implies that the two calculations – required capital and market consistent value of assets and liabilities – have to be determined consistently.

6.6.3. Going-Concern vs. Run-Off

The internal model aims at quantifying a realistic estimate of the capital needed to absorb risk over a one year time-horizon.

In most cases, the firm is in a going-concern situation at the valuation date. In most future states of the world in one year's time, the company will still be a going-concern. In a smaller set of states of the world, the firm will be in financial distress and in run-off. The models should generally take these different states of the company into account. In a run-off situation, the expenses will differ from a going-concern situation as will the cost of capital and other factors. In the case of an insurance group, restrictions on transferability of capital might apply. All these can have effects on the financial position of the firm in one year's time and should generally be taken into account in the required economic capital.

It is important that the model is very clear in distinguishing between:

- the current financial state of the company and the assumption of remaining a going concern, which determines own funds available which can buffer risks and
- future hypothetical states of the company which determine the amount of required economic capital.

The internal model should generally be able to both determine the economic balance sheet on a going concern basis but also in a run-off situation in financial distress.

²⁴ This expresses Solvency II's and many internal risk management systems' principle of "total balance sheet approach": All risks that can impact the value of a firm are "on balance sheet". Notwithstanding some liabilities are excluded from Solvency II considerations, eg the staff pension scheme, for reasons that have remained opaque.

6.6.4. Risk Measure

A risk measure supplies to a given amount of risk a currency amount. There are many potential risk measures and there is no “perfect” measure. For appropriate risk management, different risk measures can be used and the internal model should be able to calculate the required measurements. For Solvency II purposes, a value at risk on a 99.5% confidence level has to be determined. For the company’s own purposes, e.g. for capital cost allocation, coherent risk measures as for example tail value at risk (expected shortfall), may be more appropriate.

6.6.5. Dependencies

Dependencies should ideally be modelled at the level of the underlying risk factors. In contrast to that, many models besides the Solvency II Standard Formula (for example S&P, the NAIC RBC model etc.) calculate the dependencies on a highly aggregate level between capital charges for risk classes. This can be unpractical as dependencies then depend on the actual exposure of the company and may need to be recalibrated whenever the exposure changes materially.

The dependency structure can be modelled using different approaches, e.g. correlations, copulae or causal models. If possible, and supported by data and reasoning, causal approaches are preferred as then the dependencies emerge naturally from the underlying risk factors. Correlations have the advantage of being simple, but great care needs to be taken to ascertain that the implied assumptions of correlations are actually appropriate. In particular, it is difficult to allow for tail-dependencies using correlations.

The dependency between the risk factors should ideally be modelled taking into account the observations that dependencies can depend on the potential state of the world. For example, in case of a severe pandemic, financial markets can be expected to be affected while normally, mortality and financial market are not or are only weakly dependent. This comment can also serve as motivation to express dependencies with appropriate copulae, with which one can easily express “tail dependencies”, even asymmetric ones. However, such dependencies are generally difficult to calibrate reliably.

Members Groupe Consultatif's Internal Models Working Group

Country	Representative	Country	Representative
Czech Republic 	Kamil Zak	Netherlands 	Henk van Broekhoven
Denmark/ UK 	Peter Fledelius Annette Olesen	Norway 	Gyrid Johnson
Finland 	Janne Kaipio	Spain 	Regina Garcia
France 	Vincent Dupriez	Switzerland 	Philipp Keller Hans Peter Würmli (Chair)
Germany 	Thomas Schmidt (Co-Chair) Kathleen Ehrlich	UK 	Mark Chaplin David Paul
Italy 	Nino Savelli		